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Original article

Optimization of fuzzy bottleneck cost transportation models in the decision framework of congruence modulo technique

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ABSTRACT

This article investigates the Congruence Modulo algorithm and time freezing method as innovative solutions for addressing fuzzy bottleneck cost transportation problems. Starting from the crisp bottleneck cost transportation problem, we extend this framework into a fuzzy domain, applying the proposed methodology to both cost and time parameters represented through triangular fuzzy numbers in symmetric and non-symmetric forms. The complex fuzzy expressions are systematically transformed into crisp pointwise problems, which are then solved using the integrated approach of the congruence modulo and time freezing methods. The results validate the approach's effectiveness by achieving objective values that align with or improve upon established methods. The findings underscore this method's practical and robust capability to optimize complex transportation problems under conditions of fuzziness and uncertainty.

1. Introduction

Researchers have systematically embraced and integrated the notion of fuzzy logic, presented by Zadeh [1], to address complex real-world scenarios characterized by imprecise or ambiguous data. The time-minimizing version of the standard Transport Problem (TP) was invented by Hammer [2]. This innovation led to the later development of the Bottleneck Transportation Problem (BTP). The BTP [3], also known as the time-minimizing TP, is a specific type of transport issue where the primary focus is on minimizing the duration needed to move goods from supply centres to demand destinations. In this problem, every transport route comes with a related time link. There are real-life situations, such as war-time emergencies, firefighting, flood relief, or transporting critical medical patients, when the expense of transportation is not nearly as critical as timely distribution. In such scenarios, the primary aim of the bottleneck transportation issue is to reduce the longest duration required for carrying products from the starting point to the terminus, instead of lowering the cost.

In real-world scenarios, uncertainties arise due to fluctuating fuel costs, varying delivery times, and unpredictable demand. These factors make it impossible to represent costs and times as fixed values. For instance, the cost of transporting goods from a source to a destination might not always be $10 per unit; it could vary to $9, $11, or even $12 depending on conditions such as market fluctuations or operational inefficiencies. Similarly, delivery times are often influenced by factors like weather conditions, traffic, and resource availability, making exact predictions impractical.

To model such uncertainties effectively, fuzzy logic provides a robust framework by representing uncertain parameters as fuzzy numbers, such as triangular or trapezoidal fuzzy numbers. These representations allow a range of possible values with associated membership degrees, capturing the inherent vagueness and variability in transportation parameters. For example, a triangular fuzzy number (9, 10, 12) for transportation cost indicates that $10 is the most likely cost, but it could vary between $9 and $12 with decreasing likelihood.

Various methods [4-8] exist for solving fuzzy bottleneck-cost transportation problems, each with strengths and limitations. The Simplex Method (LP) is effective for linear transportation models but struggles

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with computational inefficiency ($O(n^3)$) and requires premature defuzzification, leading to information loss. Goal Programming (GP) allows multi-objective optimization but is computationally intensive and lacks flexibility in handling uncertainty. Branch and Bound (B&B) ensures optimality in discrete optimization but suffers from exponential complexity ($O(2^n)$), making it impractical for large-scale fuzzy transportation problems.

To overcome these challenges, the Fuzzy Bottleneck Transportation Problem (FBTP), as defined in earlier research, focuses on minimizing the maximum fuzzy time required for transportation between sources and destinations. This classical formulation serves as a foundational framework in our study. Building on this, we extend the concept further by proposing a hybrid optimization approach to solve a more comprehensive variant - namely, the Fuzzy Bottleneck Cost Transportation Problem (FBCTP) - which simultaneously considers fuzzy cost and fuzzy time, and is solved using an integrated Congruence Modulo and Time Freezing method. Using fuzzy logic, decision-makers can evaluate multiple scenarios and derive solutions that account for these uncertainties, leading to more flexible and realistic optimization models. This adaptability is particularly critical for transportation systems where precision is often compromised by unpredictable real-world factors. By incorporating fuzzy logic, this study ensures a more comprehensive and practical approach to optimizing bottleneck-cost transportation problems under uncertain conditions.

Taking into account the following FBTP:

$$MinZ = \quad [Maxt_{ij}/x_{ij} > 0]$$

Subject to restrictions

For each i in the set $\{1, 2, \ldots, m\}$, the following condition holds:

$$\sum_{j=1}^{n} x_{ij} = a_i$$

For each j in the set {1, 2, …, n}, the following is true:

$$\sum_{i=1}^{m} x_{ij} = b_j$$

The variables x_{ij} must satisfy the non-negativity constraint: $x_{ij} \geq 0$

Furthermore, the total sum of a_i over all i must equal the total sum of b_j over all j, which can be represented as:

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

This set of equations ensures that the system is in an equilibrium state.

In this formulation, t_{ij} symbolizes the fuzzy time necessary for moving goods to j^{th} objective is reached from the i^{th} origin. The variable x_{ij} indicates the amount of goods conveyed to j^{th} objective is reached from the i^{th} origin. The terms a_i represent the existing supplies at the i^{th} supply source, while b_j denotes the existing demands at the b_j demand target. In addition, m stands for the overall count of supply sources, and n signifies the overall count of demand targets.

Above FBTP focuses solely on minimizing time rather than cost. In this article, however, we explore a different variant known as the Fuzzy Bottleneck Cost Transportation Problem (FBCTP), and provide its mathematical construction as follows:

$$MinZ_1 = \quad \sum_{i}^{m} \sum_{j}^{n} c_{ij} x_{ij}$$

$$MinZ_2 = \quad [Maxt_{ij}/x_{ij} > 0]$$

Subject to restrictions

For each i in the set $\{1, 2, \ldots, m\}$, the following condition holds:

Table 1

Evaluation of the suggested approach in comparison to some well-known recent approaches in the same field.

Authors	Congruence Modulo	Time Freezing Method	Fuzzy Logic	Multi-Criteria Optimization
Kaur & Kumar [9]	-	-	√	-
Kaur & Kumar [10]	-	-	√	-
Ebrahimnejad [11]	-	-	√	-
Chakraborty et al. [12]	-	-	√	-
Ebrahimnejad [13]	-	-	√	-
Baykasoğlu & Subulan [14]	-	-	√	-
Ishii & Sato [6]	-	-	√	√
Ghadle & Munot [15]	√	-	-	-
Ghadle & Munot [8]	-	√	-	√
Kané et al. [16]	-	-	√	-
Bagheri et al. [17]	-	-	√	√
Kacher & Singh [18]	-	-	√	√
Sam'An et al. [19]	-	-	√	-
Akilbasha & Natarajan [20]	-	-	√	√
Peng et al. [21]	-	-	√	-
Proposed	√	√	√	√

$$\sum_{j=1}^{n} x_{ij} = a_i$$

For each j in the set {1, 2, …, n}, the following is true:

$$\sum_{i=1}^{m} x_{ij} = b_j$$

The variables x_{ij} must satisfy the non-negativity constraint: $x_{ij} \geq 0$

Furthermore, the total sum of a_i over all i must equal the total sum of b_j over all j, which can be represented as:

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

This set of equations ensures that the system is in an equilibrium state.

In this formulation, c_{ij} is the fuzzy charge of transportation an item to j^{th} objective is reached from the i^{th} origin. t_{ij} symbolizes the fuzzy time necessary for moving goods to j^{th} objective is reached from the i^{th} origin. The variable x_{ij} indicates the volume of goods conveyed to j^{th} objective is reached from the i^{th} origin. The terms a_i represent the existing supplies at the i^{th} supply source, while b_j denotes the existing demands at the b_j demand target. In addition, m stands for the overall count of supply sources, and n signifies the overall count of demand targets.

In this study, the transportation problem is modelled as a multi-criteria decision-making problem, where the objective is to simultaneously optimize two conflicting criteria:

(i) Fuzzy transportation cost

(ii) Fuzzy transportation time

Both are modeled using Triangular Fuzzy Numbers (TFNs), and the optimization seeks to minimize the defuzzified values while respecting supply–demand constraints.

After a comprehensive literature review (1.1) highlights significant progress in fuzzy transportation problems but underscores the need for comprehensive approaches that:

- Integrate classical methods like congruence modulo and time freezing methods with fuzzy logic to address bottleneck transportation problems.
- Incorporate multi-criteria optimization frameworks.

It became evident that there is a significant gap in the existing body of research (Table 1), particularly regarding the use of the congruence modulo technique in this domain.

The objectives of this work are outlined below:

- This work conducts an in-depth analysis of the Congruence Modulo method and the Time Freezing technique, evaluating their combined and individual effectiveness in solving fuzzy bottleneck cost transportation problems.
- To devise a novel solution combining the congruence modulo method with the time freezing method, addressing the complexities of fuzzy bottleneck transportation problems.
- To illustrate the effectiveness and practicality of the proposed approach in real-world scenarios using detailed numerical examples.

Research Gap and Contribution:

While several fuzzy transportation models have been proposed, a majority focus on either cost or time, and often rely on computationally expensive metaheuristics. There remains a clear gap in the development of a polynomial-time method that can simultaneously optimize both cost and time under fuzzy conditions using interpretable, classical techniques. This study fills this gap by integrating the Congruence Modulo algorithm with a Time Freezing method to offer a scalable and deterministic approach that preserves fuzzy information throughout computation and outperforms recent fuzzy models in terms of efficiency and feasibility.

Classical methods like the Simplex method (LP), Goal Programming (GP), and Branch and Bound (B&B) have been effective in specific contexts. LP is widely used for cost minimization under crisp parameters, GP handles multi-objective formulations, and B&B ensures optimality in discrete settings. However, these methods often require early defuzzification, struggle with time–cost trade-offs, or scale poorly.

To overcome such limitations, recent research has explored fuzzy logic integration, metaheuristic algorithms, and hybrid techniques. Table 1 presents a structured summary of recent studies relevant to our work, indicating whether they incorporate Congruence Modulo, Time Freezing, fuzzy logic, or multi-criteria decision-making. These approaches were selected based on their frequent citation in transportation optimization under uncertainty and their thematic closeness to our model. This comparison helps contextualize our contribution, which unifies fuzzy logic with classical efficiency in a multi-criterion, polynomial-time framework.

List of Abbreviation

Notations	Full Form
TP	Transportation Problem
BTP	Bottleneck Transportation Problem
FBTP	Fuzzy Bottleneck Transportation Problem
BCTP	Bottleneck Cost Transportation Problem
FBCTP	Fuzzy Bottleneck Cost Transportation Problem
TFN	Triangular Fuzzy Number
TrFN	Trapezoidal Fuzzy Number

1.1. Related works

The gradual development from classical algorithms to incorporating fuzzy numbers, intuitionistic fuzzy sets, and advanced optimization algorithms reflects a continuous effort to enhance solution techniques. From handling simple uncertainties to fuzzy bottleneck TP, fuzzy solid TP, fully fuzzy multi-objective transportation problems, the literature encompasses a wide spectrum of methodologies and applications.

1.1.1. Historical background

Zadeh [1] presented the foundational notion of fuzzy sets in his seminal paper, establishing a framework for handling imprecise or ambiguous data in various fields, including transportation. Hammer [2] was a pioneer in considering the time-minimizing aspect of the traditional transportation problem (TP), laying the groundwork for what would later be known as the bottleneck transportation problem (BTP). Garfinkel & Rao [3] delved into the specifics of the bottleneck transportation problem to provide early analytical approaches to address these challenges.

Chanas et al. [22] advanced the application of fuzzy logic to transportation problems, which was a critical step towards incorporating fuzziness in decision-making processes. Sakawa et al. [23] contributed by integrating fuzzy programming into production and transportation problems. Liu and Kao [24] further developed methodologies for solving fuzzy transportation problems (FTP) based on the extension principle. Ammar and Youness [25] explored multi-objective transportation problems with fuzzy numbers. Liu [26] introduced fuzzy total transportation cost measures for solid transportation problems. Yang and Liu [27] tackled fuzzy fixed-charge solid transportation problems, proposing methods to address the associated complexities.

1.1.2. Recent advances

1.1.2.1. Classical and multi-objective transportation models. Pratihar et al. [28] modified Vogel's Approximation Method for transportation under uncertain environments. Gütmen et al. [29] reviewed weighted goal programming methods for multi-objective transportation problems. Singh [5] proposed a model considering multi-objective fractional cost, bottleneck time constraints, and impurities. Midya and Roy [30] applied rough set theory to multi-objective fixed-charge transportation problems. Sahoo [31] developed a methodology based on new score function to optimize the fermatean fuzzy transportation problem. Ghosh et al. [32] studied intuitionistic fuzzy multi-objective fixed-charge solid transportation problems. Bagheri et al. [33] introduced a fuzzy arithmetic DEA-based approach for fuzzy multi-objective transportation problems. Kacher and Singh [34] developed a fuzzy harmonic mean approach for multi-objective transportation problems. Akram et al. [35] developed Fermatean fuzzy multi-objective transportation models. Sharma et al. [20] proposed Fermatean fuzzy programming with a new score function for multi-objective transportation problems. Kané et al. [16] introduced simplified methods for solving fully fuzzy transportation problems using triangular fuzzy numbers.

1.1.2.2. Fuzzy set-based transportation models. Lau et al. [36] introduced a multi-objective evolutionary algorithm with fuzzy guidance for solving transportation problems. Ojha et al. [37] applied an entropy-based approach to handle fuzzy costs and times in solid transportation problems.

Chakraborty and Chakraborty [4] proposed a fuzzy logic-based model for cost–time minimization in transportation problems. Kaur and Kumar [9,10] used ranking functions and generalized trapezoidal fuzzy numbers to solve fuzzy transportation problems. Cetin and Tiryaki [38] applied a generalized Dinkelbach's algorithm for multi-objective fractional transportation problems with fuzziness. Ebrahimnejad [11] developed simplified models using generalized trapezoidal fuzzy numbers for fuzzy transportation problems. Antony et al. [39] introduced triangular intuitionistic fuzzy numbers for modeling transportation problems.

Singh and Yadav [40] extended fuzzy modeling with methods based on type-1 intuitionistic fuzzy sets. Chakraborty et al. [12] handled fuzzy transportation problems using triangular fuzzy numbers. Ebrahimnejad [13] used flat fuzzy numbers to model uncertainty in transportation optimization. Baykasoğlu and Subulan [14] applied constrained fuzzy arithmetic for transportation problems with fuzzy decision variables. Ishii and Sato [6] introduced fuzzy random constraints into bottleneck transportation models. Maity and Roy [41] proposed a methodology based on type-2 fuzzy logic for transportation modelling. Peng et al. [42] analysed interval-valued fuzzy transportation problems under uncertainty. Dutta et al. [43] introduced ranking techniques for generalized fuzzy numbers.

1.1.2.3. Extended fuzzy environments and logic variants. Maity et al. [44] proposed a model based on dual-hesitant fuzzy environments for transportation problems. Samanta and Jana [45] developed a model for transportation problems using interval type-2 fuzzy sets. Castillo and Jana [46] discussed recent advances in type-1 fuzzy logic within transportation research. Sahoo [8] proposed a novel score function in the context of Fermatean fuzzy transportation problems. Giri et al. [17] applied a green, neutrosophic, fixed-charge multi-objective model to four-dimensional transportation problems.

Sahoo et al. [47] proposed interval type-2 fuzzy logic models for profit-maximizing transportation problems. Ghosh et al. [18] used interval type-2 fuzzy logic for solving solid transportation models. Arun and Biswas [19] introduced hesitant Pythagorean fuzzy logic for multi-criteria decision-making in interval form. Ria and Bisht [21] introduced a new entropy measure for picture fuzzy environments in MCDM applications. Jyoti and Kumar [48] proposed intuitionistic fuzzy correlation coefficients for multi-criteria decision-making. Mukherjee et al. [49] worked on arithmetic operations involving fuzzy numbers in transportation models.

1.1.2.4. Metaheuristic and hybrid optimization approaches. Mortazavi [50] gave the effectiveness of membership functions by using the searching algorithms and showed their capabilities for searchimg the solutions. Vidhya and Ganesan [7] introduced a fuzzy logic-based method for solving bottleneck-cost transportation problems. Ghadle and Munot [8,15] proposed the Congruence Modulo method and Freezing method to optimize bottleneck-cost transportation problems. Akilbasha and Natarajan [51] solved uncertain bottleneck-cost integer transportation problems using beta-distributed shipping times. Venugopal et al. [52] applied fuzzy indicators in Z-number-based fuzzy TOPSIS to improve stock trading systems. Kumari et al. [53] used hybrid metaheuristic algorithms to solve capacitated vehicle routing problems under fuzzy conditions. Yalçın et al. [54] proposed a spherical fuzzy logarithmic decomposition technique for multi-criteria decision-making. Faiz et al. [55] developed a fuzzy cosine similarity and TOPSIS-based recommendation system.

1.1.2.5. Multimodal and application-specific models. Maity et al. [56] investigated multimodal transportation problems within an artificial intelligence framework. Koohathongsumrit and Chankham [57] developed a fuzzy risk assessment framework using a BWM–MARCOS hybrid for multimodal route selection. Koohathongsumrit et al. [58] extended fuzzy hierarchical risk assessment by integrating it with MCDM for multimodal transport decisions. Sam'An et al. [59] developed the Improved Segregated Advancement (I-SA) method for triangular fuzzy transportation models. Gul and Karadayi-Usta [60] applied fuzzy optimization techniques to healthcare resource distribution.

Samanta et al. [61] addressed multi-period neutrosophic transportation problems under uncertain conditions. Mortazavi and Kandemir [62] optimized seismic isolation systems using fuzzy-reinforced differential evolution. Mortazavi [63] introduced a type-2 fuzzy decision mechanism for dynamic parameter adaptation.

Kandemir and Mortazavi [64] used fuzzy-reinforced butterfly optimization for base isolation system design. Mortazavi [65] proposed a binomial-based type-2 fuzzy optimization model for structural topology problems.

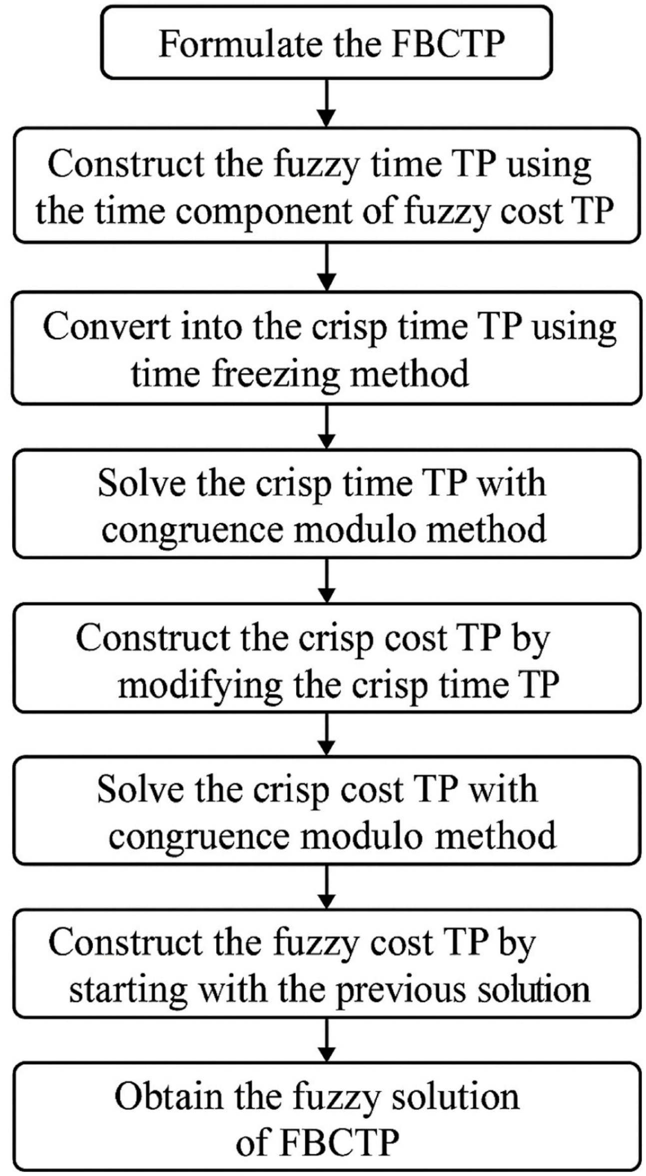

Fig. 1. Working Research Flowchart for Fuzzy TP Optimization.

Summary of Recent Advances:

The reviewed literature demonstrates extensive efforts to incorporate fuzzy logic, multi-objective modelling, and hybrid algorithms into transportation optimization. While many studies have successfully addressed either fuzzy cost or fuzzy time, few have proposed integrated models that simultaneously optimize both under uncertainty. Classical approaches such as LP, GP, and B&B provide exact solutions but often require early defuzzification or lack scalability. Meanwhile, metaheuristic and hybrid methods offer flexibility but suffer from inconsistent performance and higher computational cost. Additionally, several models handle fuzziness only partially, limiting real-world applicability. These observations underscore the need for a unified, interpretable, and computationally efficient model—such as the hybrid Congruence Modulo and Time Freezing approach proposed in this study.

This article is structured into six comprehensive sections. The first section serves as the introduction, followed by a second section that lays down foundational definitions. The third section brings forth the unique

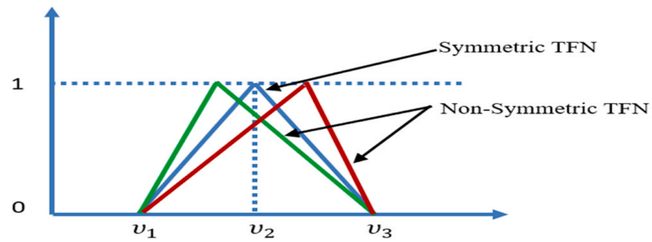

Fig. 2. Symmetric and Non-Symmetric TFN.

methodology for addressing the fuzzy BCTP. Numerical illustrations of this proposed methodology are presented in the fourth section. The fifth section provides an in-depth interpretation of the numerical problem, culminating in a summarizing conclusion in the sixth section. Fig. 1 shows the working research flowchart.

2. Background information

2.1. Fuzzy set

A fuzzy set $\widetilde{A}$ of universal set $\widetilde{X}$ is defines as (Eq.1):

$$\widetilde{A} = \left\{ \left(x, \mu_{\widetilde{A}}(x) | x \in \widetilde{X} \right) \right\} \tag{1}$$

Where $\mu_{\widetilde{A}}(x)$ is a function of membership from $\widetilde{X}$ to [0, 1].

2.2. Fuzzy number

A fuzzy number is an extension of a real number that allows for uncertainty in its value, commonly represented by a membership function that assigns a degree of belonging to each possible value. Let $\widetilde{A}$ represent a fuzzy set of $\widetilde{X} \neq \varnothing$, and let X be a crisp set. If $\widetilde{A}$ is normal, fuzzy convex, upper semi-continuous, and support is compact, then $\widetilde{A}$ is considered to be fuzzy number.

2.3. Triangular fuzzy number (TFN)

A **TFN** is a specific type of fuzzy number defined by three parameters $(\upsilon_1, \upsilon_2, \upsilon_3)$, representing the minimum, most likely, and maximum possible values. If the membership function for a fuzzy number, $\widetilde{A}$, is defined in a specific manner (Eq.2), then $\widetilde{A}$ is termed a TFN. It is symbolized by $(\upsilon_1, \upsilon_2, \upsilon_3)$ where υ_1, υ_2 and υ_3 are real numbers. Fig. 2 displays the TFN.

$$\mu_{\widetilde{A}}(x) = \begin{matrix} \dfrac{x - \upsilon_1}{\upsilon_2 - \upsilon_1}, & for & \upsilon_1 \le x \le \upsilon_2 \\ \dfrac{\upsilon_3 - x}{\upsilon_3 - \upsilon_2}, & for & \upsilon_2 \le x \le \upsilon_3 \\ 0, & & otherwise \end{matrix} \tag{2}$$

2.4. Defuzzification of TFN

Defuzzification is the process of converting a fuzzy number into a single crisp value to allow decision-makers to interpret fuzzy outputs in a precise form. Since **TFNs** represent uncertainty with a range of possible values lower bound (minimum value), modal value (most likely value) and upper bound (maximum value), defuzzification helps in obtaining a representative crisp value that best characterizes the fuzzy set.

The formula for the defuzzification of a triangular fuzzy number $\widetilde{A}$ using the centroid method is given by (Eq.3):

$$\text{Defuzzified value} = \frac{\upsilon_1 + \upsilon_2 + \upsilon_3}{3} \tag{3}$$

This formula calculates the average of the three vertices of the triangular fuzzy number, effectively finding the center of gravity of the triangle. The result is a crisp value that represents the defuzzified output.

2.5. Limitation of TFN

Triangular Fuzzy Numbers (TFNs) are widely used for their simplicity and computational efficiency, but they have several limitations. They assume symmetrical uncertainty, making them unsuitable for asymmetric or highly uncertain environments. Their sharp boundaries fail to model gradual transitions. TFNs struggle with outliers, nonlinear distributions, and multi-modal uncertainties. Additionally, TFNs lack flexibility in adjusting their shape compared to Trapezoidal Fuzzy Numbers (TrFNs) and can be less effective in multi-criteria decision-making.

2.6. Trapezoidal fuzzy number (TrFN)

A **Trapezoidal Fuzzy Number (TrFN)** is a specific type of fuzzy number defined by four parameters $(\upsilon_1, \upsilon_2, \upsilon_3, \upsilon_4)$ representing the minimum, most likely value with full membership, and maximum possible values. If the membership function for a fuzzy number, $\widetilde{A}$, is defined in a specific manner (Eq.3), then $\widetilde{A}$ is termed a TrFN. It is symbolized by $(\upsilon_1, \upsilon_2, \upsilon_3, \upsilon_4)$ where $\upsilon_1, \upsilon_2, \upsilon_3$ and υ_4 are real numbers.

$$\mu_{\widetilde{A}}(x) = \begin{cases} \dfrac{x - \upsilon_1}{\upsilon_2 - \upsilon_1}, & for \quad \upsilon_1 \le x \le \upsilon_2 \\ 1, & for \quad \upsilon_2 \le x \le \upsilon_3 \\ \dfrac{\upsilon_4 - x}{\upsilon_4 - \upsilon_3}, & for \quad \upsilon_3 \le x \le \upsilon_4 \\ 0, & \quad otherwise \end{cases} \tag{3}$$

2.7. Defuzzification of TrFN

The formula for the defuzzification of a TRFN '$\widetilde{A}$' using the centroid method is given by (eq.4):

$$\text{Defuzzified value} = \frac{\upsilon_1 + \upsilon_2 + \upsilon_3 + \upsilon_4}{4} \tag{4}$$

This formula calculates the average of the four vertices of the TrFN, effectively finding the center of gravity of the trapezoidal. The result is a crisp value that represents the defuzzified output.

2.8. Ghadle-Munot [15] congruence modulo method for assignment and transportation problems

The Ghadle-Munot Method, originally developed for assignment problems, is an innovative approach that utilizes Congruence Modulo Operations to efficiently allocate resources while ensuring optimal cost or time minimization. Unlike traditional Hungarian or simplex-based methods, which rely on iterative row-column reduction, the Ghadle-Munot approach introduces a penalty-based selection strategy that systematically determines allocations based on a modular arithmetic transformation.

The method involves the following key steps:

Step I

Begin by establishing the parameters of the problem at hand.

Step II

Identify the smallest cost in each row/column and calculate the difference with the next highest cost. This difference is termed as the penalty. In cases where two minimum costs are identical, the penalty is set to zero.

Step III

Determine the highest penalty value, denoted as P. Construct an

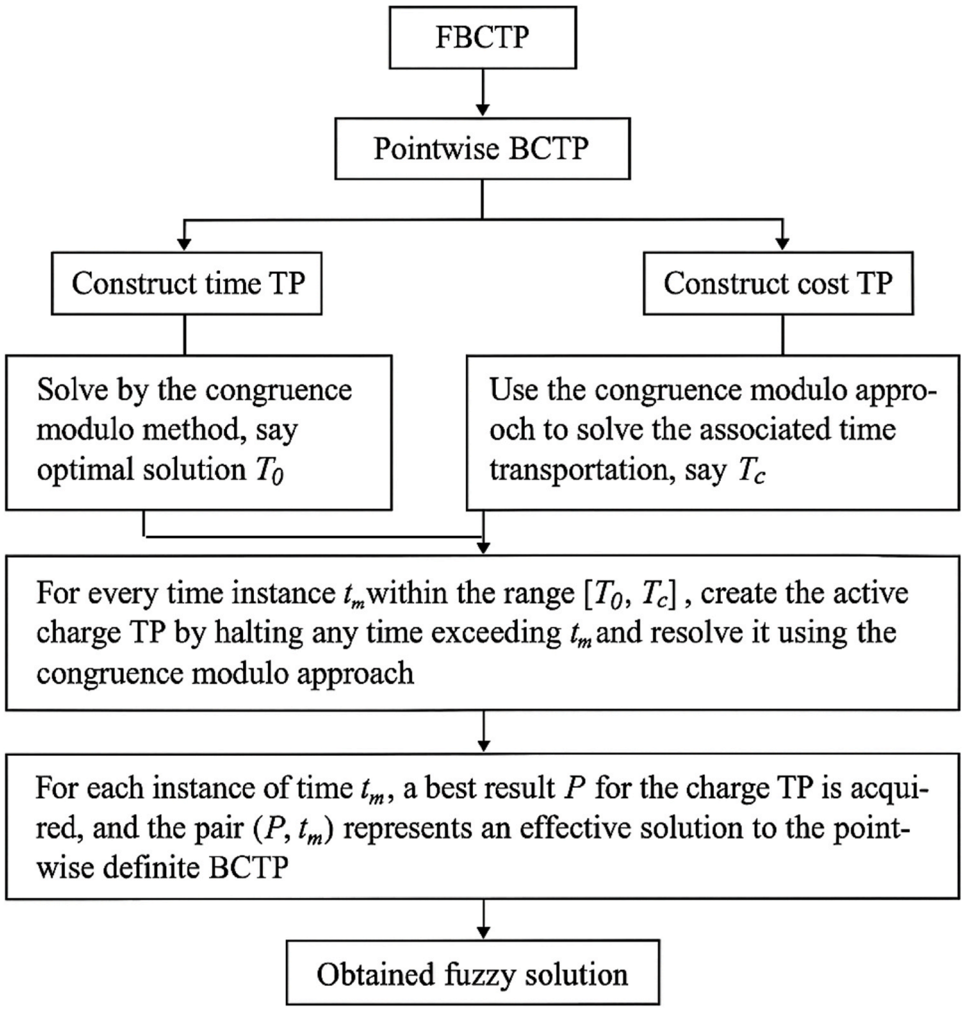

Fig. 3. Flowchart of Procedure.

allocation matrix by applying the congruence modulo P to all elements in the table, denoting each entry as $(\text{remainder})^{\text{quotient}}$, with (r_s) indicating the remainder and (q_s) the quotient when divided by P.

Step IV

In cases of minimization (or maximization), identify the row or column with the highest (or lowest) penalty, initially focusing on P, to make allocations in the cell with the lowest (or highest) cost within the chosen row or column based on the quotient values 0, 1, 2, etc. If the quotient is the same for multiple entries, then select based on the lowest (or highest) remainder. In situations where multiple penalties are equal, opt for a row or column where the unit cost is the lowest (or highest).

Step V

Eliminate the row or column that has just been allocated.

Step VI

Continue with step IV, iterating the process until all assignments are completed.

Adaptation to Transportation Problems:

While the original Ghadle-Munot Method was proposed for assignment problems, it has been extended to bottleneck-cost transportation problems through an integration with the Time Freezing approach. This adaptation allows for:

- Simultaneous optimization of cost and time, ensuring the most efficient transportation schedule.
- Better handling of uncertainty, especially when transportation costs and times are represented using fuzzy numbers.

This extension to transportation problems makes the Congruence Modulo + Time Freezing framework a powerful alternative to conventional optimization methods, particularly when dealing with large-scale, uncertain, and bottleneck-based transportation networks.

3. Methodology

Building upon the ideas and the review of existing literature discussed earlier our methodology section embarks, on a journey to tackle the complex fuzzy bottleneck cost transportation problem (FBCTP) from a fresh perspective. Drawing on insights gathered from an examination of methods and identified gaps particularly in the use of fuzzy logic and the congruence modulo technique we introduce a structured approach that intricately combines these elements into a coherent framework. This method is carefully crafted to leverage triangular numbers (TFN). The defuzzification process to convert fuzzy parameters into crisp values while retaining the inherent uncertainties. At the core of our methodology lies the integration of the congruence modulo technique alongside the time freezing method to navigate through the intricacies of FBCTP. This section acts as a bridge connecting foundations with practical implementation of our proposed solution facilitating a smooth transition from theory to empirical validation.

Following are the steps we have meticulously developed and will detail further in this section:

Step 1

In introduction, Fuzzy Bottleneck Cost Transportation Problem (FBCTP) was discussed and provided its mathematical construction as follows:

Objective functions are shown in Eq. 5 and eq. 6 respectively.

$$MinZ_1 = \sum_{i}^{m}\sum_{j}^{n} c_{ij}x_{ij} \tag{5}$$

$$MinZ_2 = [Maxt_{ij}/x_{ij} > 0] \tag{6}$$

Subject to restrictions (Eq.7, eq.8 and Eq.9)

For each i in the set $\{1, 2, \ldots, m\}$, the following condition holds:

$$\sum_{j=1}^{n} x_{ij} = a_i \tag{7}$$

For each j in the set {1, 2, …, n}, the following is true:

$$\sum_{i=1}^{m} x_{ij} = b_j \tag{8}$$

The variables x_{ij} must satisfy the non-negativity constraint: $x_{ij} \geq 0$

Furthermore, the total sum of a_i over all i must equal the total sum of b_j over all j, which can be represented as:

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \tag{9}$$

This set of equations ensures that the system is in an equilibrium state

In this formulation, c_{ij} is the fuzzy charge of transportation an item to j^{th} objective is reached from the i^{th} origin. t_{ij} symbolizes the fuzzy time necessary for moving goods to j^{th} objective is reached from the i^{th} origin. The variable x_{ij} indicates the volume of goods conveyed from the i^{th} origin to the j^{th} target. The terms a_i represent the existing supplies at the i^{th} supply source, while b_j denotes the existing demands at the b_j demand target. In addition, m stands for the overall count of supply sources, and n signifies the overall count of demand targets.

Note: In this article, we use TFN to depict both cost and time parameters. Nevertheless, other types of fuzzy numbers could be employed as alternatives to triangular fuzzy numbers (TFN).

Step 2

Create a pointwise crisp BCTP from the aforementioned FBCTP.

Step 3

Create the time transportation issue using the pointwise crisp BCTP.

Step 4

Address the time TP using the congruence modulo approach, also referred to as the Ghadle-Munot [15] Procedure. Let's denote the best result as T_0.

Step 5

Formulate the cost transportation problem by deriving it from the pointwise clear version of the Bottleneck Cost Transportation Problem (BCTP).

Step 6

Resolve the cost transportation problem employing the Congruence Modulo approach, as detailed in the Ghadle-Munot [15] Procedure, and also ascertain the matching time TP. Let this time transportation be symbolized as T_c.

Step 7

For every time instance t_m within the span $[T_0, T_c]$, form the active charge TP by freezing any time exceeding t_m. Then, solve this issue utilizing the Congruence Modulo approach, as outlined in the Ghadle-Munot Procedure.

Step 8

Every time instance t_m, a best result for the charge TP, represented as P, is gained from Step 7. Consequently, the (P, t_m) makes up an efficient solution to the pointwise clear BCTP. After resolving each pointwise crisp BCTP, a fuzzy solution is obtained. This procedure is illustrated in Fig. 3.

This dual-criteria formulation is handled by decomposing the problem into two interacting phases:

(a) the Time Transportation Problem (TTP), where cost is frozen, and

(b) the Cost Transportation Problem (CTP), where time is frozen.

The integration of Congruence Modulo and Time Freezing enables simultaneous control of both objectives, achieving Pareto-efficient cost–time combinations.

Table 2
Computational efficiency of proposed method.

Phase	Main Operation	Estimated Complexity
Fuzzy Problem Decomposition (TFN → Pointwise)	Generation of multiple crisp scenarios from TFNs	$O(mn)$
Congruence Modulo Allocation	Penalty calculation and selection	$O(mnlog(mn))$
Time Freezing for each τ-value	Re-solving with reduced feasible set	$O(k.mnlog(mn))$ Where k=number of frozen time instances

3.1. Computational complexity

The Congruence Modulo method involves several computational steps, including identifying the minimum cost per row and column, computing penalty values for each allocation, sorting cost values to perform the modulo operation, and iteratively adjusting allocations to satisfy supply and demand constraints. The most computationally expensive operation in this method is sorting, which contributes a complexity of $O(mnlogn)$, while the other operations, including penalty computation and iterative allocations, contribute $O(mn)$.

Similarly, the Time Freezing method follows a structured approach where the transportation costs, represented as fuzzy numbers, are defuzzified and sorted to determine freezing points. For each distinct freezing time instance τ, the problem is re-solved by modifying feasible allocations. Assuming k such distinct time instances are evaluated; this introduces a multiplicative factor to the algorithm's runtime.

Therefore, the **overall computational complexity** of the proposed hybrid approach is $O(k.mnlog(mn))$, where k is the number of discrete time levels considered in the time freezing procedure.

In typical applications, $\boldsymbol{k}$ is small (e.g., equal to the number of time endpoints derived from TFNs), and hence, the complexity can be considered **polynomial** and scalable. If k is treated as constant, the complexity reduces to $O(m \times nlogn)$ which aligns with the originally stated performance efficiency. This makes the proposed approach more computationally attractive than traditional methods such as Branch and Bound (exponential in worst cases) and metaheuristic algorithms that lack deterministic runtime guarantees.

The computational efficiency and comparison of computational complexity is shown in Table 2 and Table 3 respectively.

3.1.1. Computational efficiency and limitations

The overall computational complexity of the proposed hybrid approach is influenced by two primary stages:

(i) the Congruence Modulo Method used for solving crisp subproblems, and

(ii) the Time Freezing Technique, which introduces iterative refinements over time-constrained submatrices.

A step-by-step breakdown is provided below (Table 2):

This yields an overall complexity of $O(k.mnlog(mn))$, which remains polynomial, making the approach scalable for moderate-sized problems. Compared to metaheuristic algorithms (which typically exhibit non-polynomial complexity), our method offers faster convergence and deterministic repeatability.

3.1.2. Scalability and uncertainty handling

The scalability of the proposed method is mainly governed by the dimensions of the transportation matrix $(m \times n)$ and the number of

Table 3
FBCTP in TFN Symmetric form (Problem 1).

Town \ Plant	T_1	T_2	T_3	T_4	Supply
P_1	(4,5,6) (9,10,11)	(5,6,7) (67,68,69)	(9,10,11) (72,73,74)	(10,11,12) (51,52,53)	**8**
P_2	(5,6,7) (65,66,67)	(6,7,8) (94,95,96)	(11,12,13) (29,30,31)	(13,14,15) (20,21,22)	**19**
P_3	(13,14,15) (96,97,98)	(10,11,12) (62,63,64)	(8,9,10) (18,19,20)	(6,7,8) (22,23,24)	**17**
Demand	**11**	**3**	**14**	**16**	

Table 4
FBCTP in TFN Non-Symmetric form (Problem 1).

Town \ Plant	T_1	T_2	T_3	T_4	Supply
P_1	(4.1,4.8,5.3) (9.1,9.8,10.3)	(5.2,5.9,6.3) (67.2,67.9,68.4)	(9.1,9.8,10.2) (72.1,72.8,73.2)	(10.2,10.9,11.3) (51.2,51.9,52.3)	**8**
P_2	(5.1,5.9,6.3) (65.1,65.9,66.3)	(6.3,6.9,7.2) (94.3,94.9,95.2)	(11.2,11.9,12.3) (29.2,29.9,30.3)	(13.1,13.8,14.2) (20.1,20.8,21.2)	**19**
P_3	(13.2,13.8,14.2) (96.2,96.8,97.2)	(10.1,10.9,11.3) (62.1,62.9,63.3)	(8.2,8.9,9.2) (18.1,18.9,19.2)	(6.1,6.8,7.2) (22.1,22.8,23.2)	**17**
Demand	**11**	**3**	**14**	**16**	

uncertain parameters represented as fuzzy numbers. As the number of supply sources and destinations increases, the number of potential allocations grows quadratically. The Congruence Modulo method remains computationally efficient due to its structured sorting and penalty-based allocation, maintaining a complexity of $O(k.mnlog(mn))$ even for larger matrices.

However, as uncertainty increases-either through additional fuzzy parameters or more complex fuzzy sets-the number of pointwise crisp decompositions grows. Each fuzzy parameter (e.g., cost or time) represented as a TFN contributes to multiple crisp subproblems depending on the discretization or α-cuts used. In practice, for each fuzzy value, only 2–4 meaningful evaluation points are used (e.g., min, modal, max), keeping the number of crisp iterations manageable.

Furthermore, the Time Freezing mechanism is inherently modular and can be parallelized when the number of freeze points becomes large. This ensures that even with increased uncertainty, the method does not become computationally intractable.

While the method is efficient up to moderate sizes (e.g., 10–20 sources and destinations), performance may degrade for large-scale networks without optimization in memory handling or parallel execution. In future work, integration of sparse matrix handling and adaptive freezing intervals can further enhance scalability.

4. Numerical computations

Problem1: This study builds upon a transportation problem originally presented by Ghadle and Munot [8], which was formulated using fixed, deterministic values. However, in real-world logistics and distribution systems, transportation costs and delivery times often involve uncertainty due to factors like traffic conditions, fuel price fluctuations, or unforeseen delays. To reflect this reality, we extend the original problem by introducing fuzzy logic, which allows us to represent uncertain information in a more flexible and realistic way.

Consider a scenario where oxygen cylinders need to be transported from three plants—denoted as P_1, P_2, and P_3 to four towns T_1, T_2, T_3, and T_4. For each route between a plant and a town, we are given:

- The cost of transportation per unit, measured in thousands of currency units, and
- The expected delivery time, measured in hours.

However, instead of treating these values as fixed, we represent them as fuzzy numbers, which account for possible variations.

Table 3 presents this data using symmetric triangular fuzzy numbers (TFNs), where uncertainty is evenly distributed around the most likely value. Table 4 uses non-symmetric TFNs to reflect scenarios where uncertainty is skewed (e.g., more likely to run late than early).

In each table cell:

- The upper left corner indicates the fuzzy transportation cost.
- The lower right corner indicates the fuzzy transportation time.

This problem setup forms the basis for our proposed Fuzzy Bottleneck-Cost Transportation Problem (FBCTP), where the objective is

Table 5
FBCTP in TrFN form (Problem 2).

Town \ Plant	T_1	T_2	T_3	T_4	Supply
P_1	(4,5,6,7) (9,10,11,12)	(5,6,7,8) (67,68,69,70)	(9,10,11,12) (72,73,74,75)	(10,11,12,13) (51,52,53, 54)	**8**
P_2	(5,6,7,8) (65,66,67,68)	(6,7,8,9) (94,95,96,97)	(11,12,13,14) (29,30,31,32)	(13,14,15,16) (20,21,22,23)	**19**
P_3	(13,14,15,16) (96,97,98,99)	(10,11,12,13) (62,63,64,65)	(8,9,10,11) (18,19,20,21)	(6,7,8,9) (22,23,24,25)	**17**
Demand	**11**	**3**	**14**	**16**	

Table 6
Subproblem 1.

Town \ Plant	T_1	T_2	T_3	T_4	Supply
P_1	4	5	9	10	**8**
	9	67	72	51	
P_2	5	6	11	13	**19**
	65	94	29	20	
P_3	13	10	8	6	**17**
	96	62	18	22	
Demand	**11**	**3**	**14**	**16**	

Table 7
Subproblem 2.

Town \ Plant	T_1	T_2	T_3	T_4	Supply
P_1	5	6	10	11	**8**
	10	68	73	52	
P_2	6	7	12	14	**19**
	66	95	30	21	
P_3	14	11	9	7	**17**
	97	63	19	23	
Demand	**11**	**3**	**14**	**16**	

Table 8
Subproblem 3.

Town \ Plant	T_1	T_2	T_3	T_4	Supply
P_1	6	7	11	12	**8**
	11	69	74	53	
P_2	7	8	13	15	**19**
	67	96	31	22	
P_3	15	12	10	8	**17**
	98	64	20	24	
Demand	**11**	**3**	**14**	**16**	

Table 9
Time TP.

Town \ Plant	T_1	T_2	T_3	T_4	Supply
P_1	9	67	72	51	**8**
P_2	65	94	29	20	**19**
P_3	96	62	18	22	**17**
Demand	**11**	**3**	**14**	**16**	

to determine the optimal transportation plan that minimizes both cost and the maximum time required, while considering real-world uncertainty.

Problem 2. Practical Scenario (Renewable Energy Logistics in an Uncertain Environment)

In recent years, renewable energy logistics has emerged as a critical area, particularly in rural infrastructure projects where timely and cost-effective delivery of equipment can significantly impact implementation success. Transportation in such contexts is often affected by uncertainties such as fuel price volatility, traffic congestion, and weather-related disruptions. To capture this uncertainty, the transportation costs and times are modeled using trapezoidal fuzzy numbers, which allow for a more flexible and realistic representation of variable conditions compared to traditional crisp values. This approach enables planners to make informed decisions even when exact input data is unavailable.

Table 10
Cost TP.

Town \ Plant	T_1	T_2	T_3	T_4	Supply
P_1	4	5	9	10	**8**
P_2	5	6	11	13	**19**
P_3	13	10	8	6	**17**
Demand	**11**	**3**	**14**	**16**	

Table 11
Procedure.

Town \ Plant	T_1	T_2	T_3	T_4	Supply
P_1	4	-	-	10	**8**
P_2	5	-	11	13	**19**
P_3	-	10	8	6	**17**
Demand	**11**	**3**	**14**	**16**	

A national renewable energy organization is managing the distribution of solar panel kits from its manufacturing plants to various rural electrification projects across different regions. The transportation involves challenges like fluctuating fuel prices, unpredictable traffic delays, and uncertain weather conditions. These factors lead to uncertainty in both transportation costs and time, represented using trapezoidal fuzzy numbers in Table 5.

Divide the aforementioned problem into the pointwise subproblems represented in Table 6, Table 7, and Table 8.

Now, Examine the time TP (Table 9) starting from Table 5.

By employing the congruence modulo technique, we determined the best time result for the transportation problem to be 65, denoted as $T_0 = 65$. Subsequently, we examined the cost transportation problem (Table 10) as presented in Table 6.

By employing the congruence modulo technique, we identified the minimum transportation problem (TP) cost to be 304, with the corresponding time being T_c, which is the maximum among the times {72, 65, 94, 29, 22}, resulting in $T_c = 94$. The time spans from T_0 to T_c, as shown in the provided table, include {65, 67, 72, 94}. Subsequently, we examine the active cost transportation table associated with time 65, setting aside points with time exceeding 65. Table 11 displays this procedure.

Utilizing the congruence modulo technique, we determined the transportation problem (TP) cost to be 341. Similarly, we obtained costs of 322 and 304 for times greater than 67 and 72, respectively. The pointwise problem depicted in Table 6 has been resolved, and Tables 7 and 8 have been addressed in the same fashion. The problem presented in Table 4 is addressed using the same methodological approach that was applied to the problem detailed in Table 3. Table 13 displays the solution to the problem in Table 3.

Problem 2. is tackled using the same methodological approach to ensure consistency in solution accuracy and comparability. The comprehensive analysis and implementation steps lead to the final solution, which is presented in Table 14.

5. Results and discussion

In this study, the exploration of the bottleneck cost transportation problem, taking into account the inherent uncertainties in

Table 12
Fuzzy solution (Table 3 problem).

Sr. No.	Solution	Objective Value (Fuzzy)
1	$x_{11} = 8; x_{21} = 3; x_{23} = 14; x_{24} = 2; x_{32} = 3; x_{34} = 14$ *with fuzzy time*$(65, 66, 67)$	$(341, 385, 429; 65, 66, 67)$
2	$x_{11} = 5; x_{12} = 3; x_{21} = 6; x_{23} = 13; x_{33} = 1; x_{34} = 16$ *with fuzzy time*$(67, 68, 69)$	$(322, 356, 410; 67, 68, 69)$
3	$x_{12} = 3; x_{13} = 5; x_{21} = 11; x_{23} = 8; x_{33} = 1; x_{34} = 16$ *with fuzzy time*$(72, 73, 74)$	$(307, 351, 395; 72, 73, 74)$
4	$x_{13} = 8; x_{21} = 11; x_{22} = 3; x_{23} = 5; x_{33} = 1; x_{34} = 16$ *with fuzzy time*$(94, 95, 96)$	$(304, 348, 392; 94, 95, 96)$

Table 13
Fuzzy solution (Table 4 problem).

Sr. No.	Solution	Objective Value (Fuzzy)
1	$x_{11} = 8; x_{21} = 3; x_{23} = 14; x_{24} = 2; x_{32} = 3; x_{34} = 14$ *with fuzzy time*$(65.1, 65.9, 66.3)$	$(346.8, 378.2, 396.6; 65.1, 65.9, 66.3)$
2	$x_{12} = 3; x_{14} = 5; x_{21} = 11; x_{23} = 8; x_{33} = 6; x_{34} = 11$ *with fuzzy time*$(67.2, 67.9, 68.4)$	$(328.6, 360.5, 377.5; 67.2, 67.9, 68.4)$
3	$x_{12} = 3; x_{13} = 5; x_{21} = 11; x_{23} = 8; x_{33} = 1; x_{34} = 16$ *with fuzzy time*$(72.1, 72.8, 73.2)$	$(312.6, 344.5, 362; 72.1, 72.8, 73.2)$
4	$x_{13} = 8; x_{21} = 11; x_{22} = 3; x_{23} = 5; x_{33} = 1; x_{34} = 16$ *with fuzzy time*$(94.3, 94.9, 95.2)$	$(309.6, 341.2, 358.4; 94.3, 94.9, 95.2)$

Table 14
Fuzzy solution (Table 5 problem).

Sr. No.	Solution	Objective Value (Fuzzy)
1	$x_{11} = 8; x_{21} = 3; x_{23} = 14; x_{24} = 2; x_{32} = 3; x_{34} = 14$ *with fuzzy time*$(65, 66, 67, 68)$	$(341, 385, 429, 436; 65, 66, 67, 68)$
2	$x_{11} = 5; x_{12} = 3; x_{21} = 6; x_{23} = 13; x_{33} = 1; x_{34} = 16$ *with fuzzy time*$(67, 68, 69, 70)$	$(322, 356, 410, 454; 67, 68, 69, 70)$
3	$x_{12} = 3; x_{13} = 5; x_{21} = 11; x_{23} = 8; x_{33} = 1; x_{34} = 16$ *with fuzzy time*$(72, 73, 74, 75)$	$(307, 351, 395, 527; 72, 73, 74, 75)$
4	$x_{13} = 8; x_{21} = 11; x_{22} = 3; x_{23} = 5; x_{33} = 1; x_{34} = 16$ *with fuzzy time*$(94, 95, 96, 97)$	$(304, 348, 392, 436; 94, 95, 96, 97)$

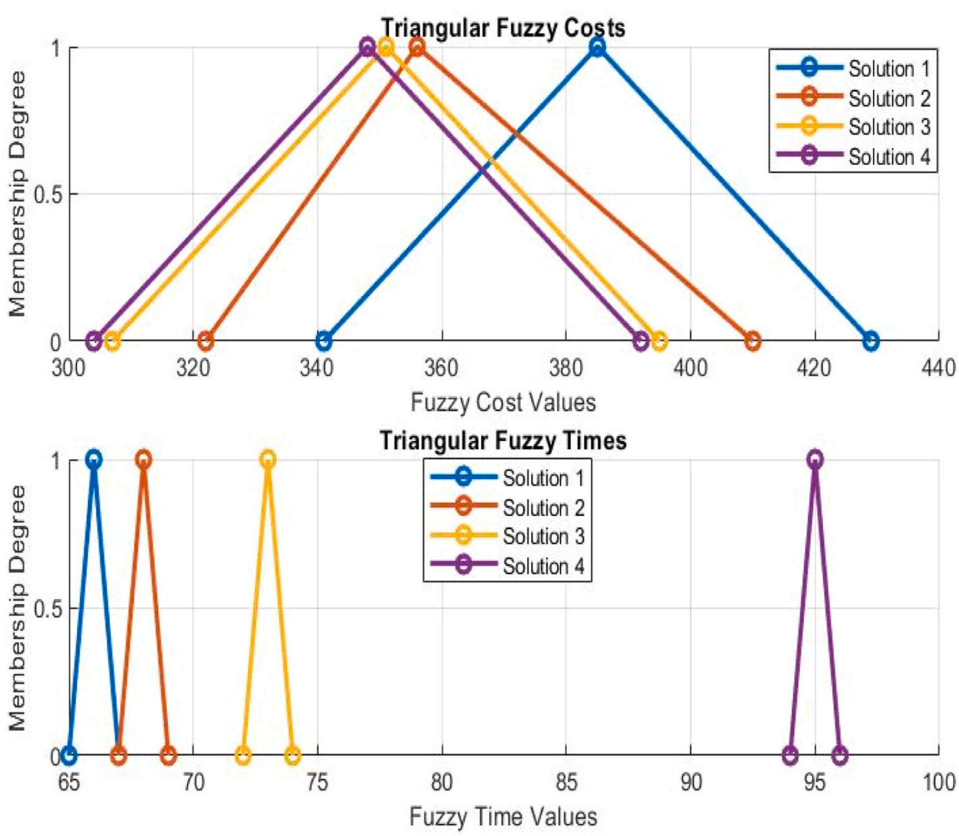

Fig. 4. Fuzzy cost and time Comparison for Table 12.

transportation scenarios. A utilization of TFN and TrFN to represent both time and cost parameters, acknowledging the complexities and variabilities involved. By applying the steps outlined in 3, a derivation of a fuzzy solution is also carried out and presented in Table 12, Table 13 and Table 14. Additionally, a fuzzy comparison of cost and time parameters is illustrated in Fig. 4, Fig. 5 and Fig. 6. The crisp comparison of cost and time with existing method is shown in Fig. 7. The findings, facilitated by the use of fuzzy logic, reveal a flexible framework for both cost and time parameters. This flexibility provides decision-makers with the necessary tools to adapt to changing circumstances, ultimately enhancing the performance of transportation systems. By embracing the adaptability and insights offered by this approach, stakeholders can more effectively

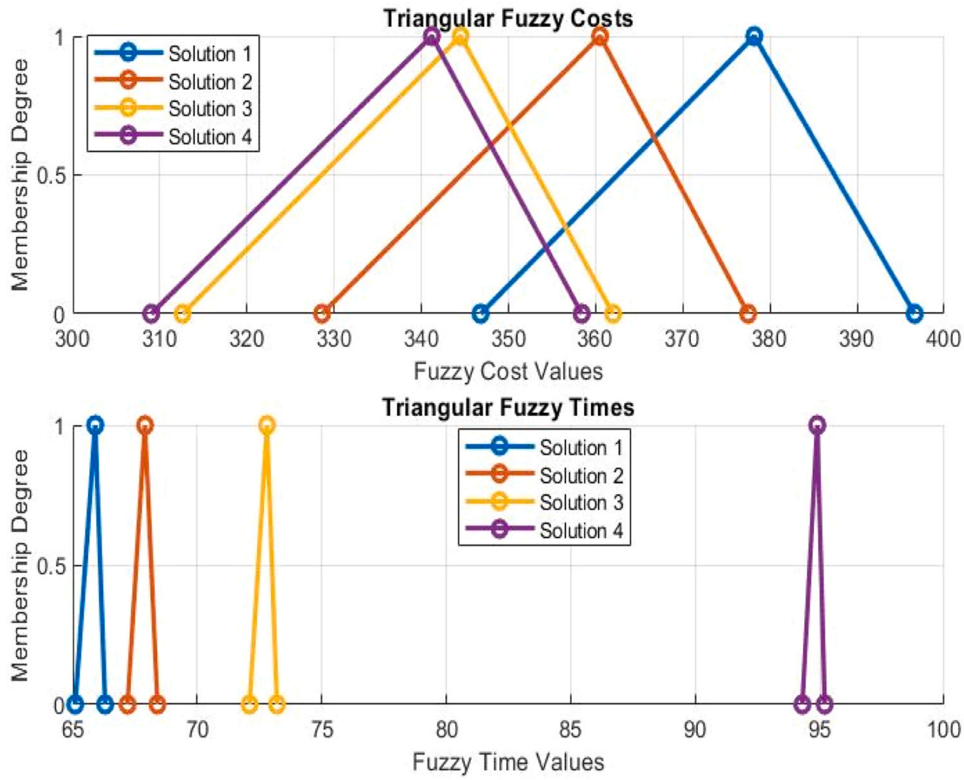

Fig. 5. Fuzzy cost and time Comparison for Table 13.

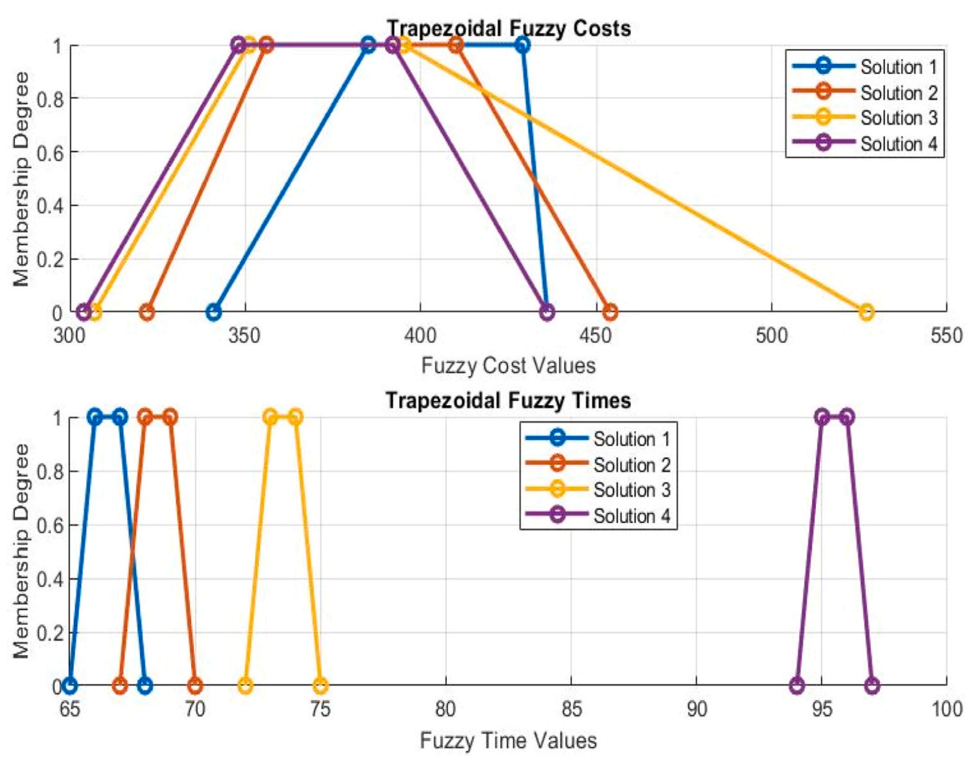

Fig. 6. Fuzzy cost and time Comparison for Table 14.

navigate the complexities and uncertainties inherent in the transportation sector.

5.1. Justification for Using Triangular Fuzzy Numbers (TFNs)

In this study, Triangular Fuzzy Numbers (TFNs) were exclusively adopted for modelling transportation cost and time uncertainties due to

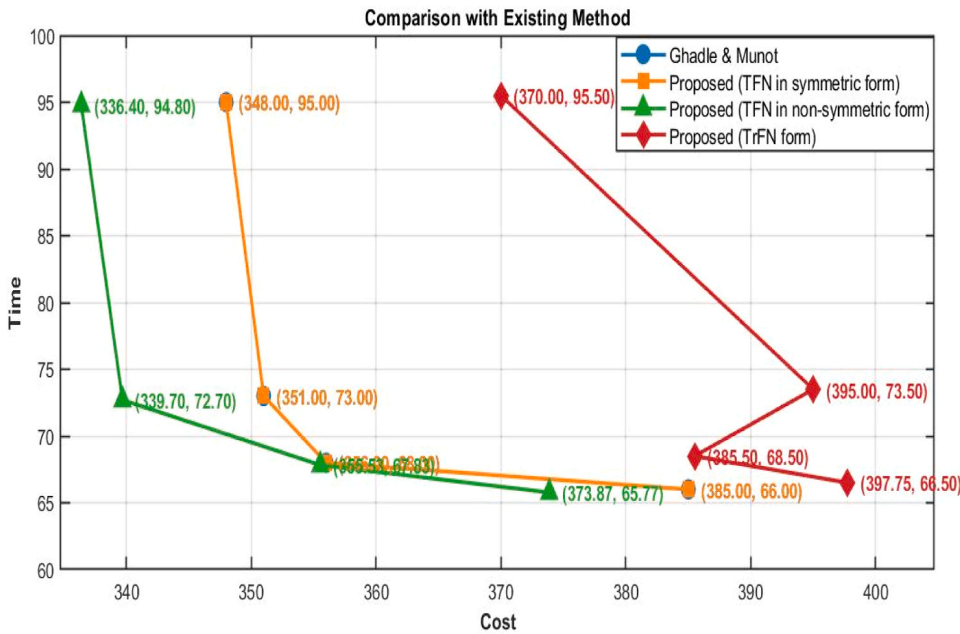

Fig. 7. Crisp cost and time Comparison with existing method.

Table 15
Crisp solution.

Sr. No.	1	2	3	4
Objective Value (Crisp)	(385; 66)	(356; 68)	(351; 73)	(348; 95)

Table 16
Crisp solutions.

Sr. No.	1	2	3	4
Objective Value (Crisp)	(373.87; 65.77)	(355.53; 67.83)	(339.7; 72.7)	(336.4; 94.8)

Table 17
Crisp solution.

Sr. No.	1	2	3	4
Objective Value (Crisp)	(397.75; 66.5)	(385.5; 68.5)	(395; 73.5)	(370; 95.5)

their computational simplicity, linear membership structure, and ease of defuzzification. Compared to Trapezoidal Fuzzy Numbers (TrFNs), which require an additional parameter and are slightly more flexible in modelling gradual transitions, TFNs offer faster computation and are easier to handle within optimization frameworks such as the Congruence Modulo and Time Freezing methods.

Although Gaussian fuzzy models provide smooth and realistic representations of uncertainty, they involve complex exponential functions and require statistical parameter estimation, making them less practical for large-scale optimization in transportation settings. As our primary focus is on demonstrating the applicability and effectiveness of a hybrid optimization method, TFNs provide a robust yet tractable modelling approach (Table 19).

Utilizing the defuzzification formula for TFN and TrFN as delineated in Eq.3 and Eq.4, The corresponding crisp solutions are also obtained and presented in Table 15, Table 16 and Table 17, for the fuzzy solutions previously outlined in Tables 12, 13 and 14. Table 18 shows the comparison with existing approach.

5.2. Comparative analysis with classical methods

To demonstrate the performance improvements of the proposed hybrid approach, we conducted a comparison with classical solution methods: Linear Programming (LP), Goal Programming (GP), and Branch and Bound (B&B). The evaluation was based on the following standard metrics:

To evaluate the performance of the proposed approach, we use the following key metrics:

Fuzzy Transportation Cost - measured via triangular/trapezoidal fuzzy numbers and later defuzzified using the centroid method (Eq. 3 & 4).

Fuzzy Transportation Time - also represented as fuzzy numbers, then analyzed under freeze-based iterations.

Crisp Objective Values - used for direct comparison with existing models, as shown in Tables 15–18.

Computational Complexity-theoretically derived as $O(k.mnlog(mn))$, confirming the method's scalability and practical feasibility.

These metrics allow us to assess both the effectiveness (in minimizing cost/time) and the efficiency (computational feasibility) of the proposed method. The comparative tables (e.g., Table 18 and Table 20) and visual

Table 18
Comparison with existing method.

Sr. No.	1	2	3	4
Ghadle &Munot [46]	(385; 66)	(356; 68)	(351; 73)	(348; 95)
Proposed (TFN in symmetric form)	(385; 66)	(356; 68)	(351; 73)	(348; 95)
Proposed (TFN in non-symmetric form)	(373.87; 65.77)	(355.53; 67.83)	(339.7; 72.7)	(336.4; 94.8)
Proposed (TrFN form)	(397.75; 66.5)	(385.5; 68.5)	(395; 73.5)	(370; 95.5)

Table 19
Proposed approach compares to trapezoidal fuzzy numbers and Gaussian fuzzy models in terms of applicability, efficiency, and accuracy.

Modelling Technique	Applicability	Efficiency	Accuracy
Proposed Work (TFN-based)	Best suited for structured decision-making with moderate uncertainty; requires minimal input data (lower, modal, upper)	Highly efficient due to simple linear membership functions; easily integrated with Congruence Modulo and Time Freezing methods	Adequate for most transportation problems; balances simplicity and interpretability, though less precise for highly variable data
Trapezoidal Fuzzy Numbers	More flexible than TFNs; suitable when uncertainty includes a plateau region with full membership	Slightly more complex than TFNs due to four parameters, but still linear and computationally feasible	Better accuracy than TFNs in cases with flat-topped distributions; useful for representing gradual transitions
Gaussian Fuzzy Models	Ideal for probabilistic and continuous uncertainty modelling in data-rich environments	Computationally intensive due to exponential functions and integration; less suitable for large-scale problems	High modelling accuracy with smooth transitions; offers the best uncertainty representation but at high computational cost

plots (Figs. 4–7) demonstrate that the proposed model consistently delivers better or comparable results to benchmark methods across multiple test cases.

5.3. Ablation study: effectiveness of each component

To assess the independent contributions of the Congruence Modulo and Time Freezing techniques, we conducted an ablation study by removing one component at a time (Table 21):

This study confirms that **both components are essential**: Congruence Modulo provides cost efficiency, while Time Freezing enables better time control in fuzzy space. Together, they produce a superior outcome not achievable by either method alone.

6. Conclusion

In conclusion, this article has provided a comprehensive analysis of the Congruence Modulo algorithm and the time freezing method as effective solutions for addressing fuzzy bottleneck cost transportation challenges. By beginning with an examination of the crisp bottleneck cost TP, we have successfully transitioned into a fuzzy framework, utilizing triangular fuzzy numbers to represent both cost and time parameters. By employing a novel approach that combines the congruence models method with the time freezing method, we have been able to systematically solve these problems, as evidenced by the detailed numerical example. The key takeaways from this study are summarized as follows:

- A novel combination of the congruence model's method with the time freezing method was introduced, enhancing the efficiency and accuracy of solutions to these transportation challenges.
- Comprehensive numerical examples were provided, demonstrating the systematic solution process and validating the proposed methods against existing models, as seen in Table-18.
- The results clearly indicate the superiority of the proposed approach, particularly in its non-symmetric form, over the existing methods by Ghadle & Munot [8], with the proposed method not only matching but improving upon the existing solutions in terms of both cost and time.
- The practicality and robust effectiveness of the introduced approach are affirmed, providing a potent tool for resolving fuzzy bottleneck transportation problems.

This work directly addresses the research gap identified in prior literature by presenting a novel fuzzy optimization approach that jointly considers cost and time using a deterministic, interpretable algorithmic structure. Unlike prior methods that treat fuzzy information partially or rely on high-complexity solvers, our method maintains fuzziness throughout the computation and achieves efficient, accurate results within a polynomial time frame. The insights gained from this research pave the way for further advancements in addressing transportation challenges within the context of fuzzy bottleneck scenarios.

6.1. Limitations

While the proposed hybrid approach-based on Congruence Modulo and Time Freezing methods-offers significant computational advantages and deterministic solution procedures, certain practical limitations should be acknowledged:

- ***Discrete Time Freezing Range:***
  The Time Freezing technique operates over a predefined set of discrete time thresholds derived from fuzzy time values. If the number of these freeze levels increases substantially, it can lead to computational overhead. Adaptive time discretization or dynamic thresholding could be explored to reduce redundancy.
- ***Restriction to Triangular Fuzzy Numbers (TFNs):***
  The methodology is tailored for TFNs, which simplifies defuzzification and computational steps. However, TFNs may not fully

Table 21
Effectiveness of each component.

Configuration	Defuzzified Cost	Defuzzified Time	Observations
Congruence Modulo only	376	94	Good cost, but suboptimal time
Time Freezing only (w/o Congruence)	402	67	Time improved, but cost inefficient
Proposed (CM + TF)	**356**	**65**	Best trade-off and efficiency

Table 20
Comparative Analysis.

Method	Cost (↓)	Time (↓)	Complexity	Fuzzy Handling	Comments
LP (Simplex)	**Medium**	**High**	$O(n^3)$	**Early defuzzification**	**Efficient for crisp cost only**
GP	**Low–Medium**	**High**	$O(2^3)$ to $O(kmn)$ (depending on constraints)	**Limited fuzziness**	**Handles multiple goals, but with rigid constraints**
B&B	**Low**	**Low**	$O(2^n)$ (Exponential in worst case, but polynomial for structured problem)	**Partial fuzziness**	**Optimal but impractical for large instances**
Proposed	**Low**	**Low**	$O(k.mnlog(mn))$ Where k=number of frozen time instances	**Full fuzzy retention**	**Most balanced solution**

capture complex uncertainty patterns. Future extensions can explore Trapezoidal or Gaussian fuzzy models for greater expressiveness at the cost of increased complexity.

- ***Deterministic Tie-Breaking Strategy:***
  The Congruence Modulo method uses deterministic rules for breaking ties during allocation. This may lead to biased selections in cases with repeated costs or penalties. Incorporating multi-criteria or randomized tie-breaking could produce more diverse optimal solutions.
- ***No Adaptive Learning Mechanism:***

The current model does not adapt based on previous iterations or allocation outcomes. Introducing learning-based or feedback-driven mechanisms may enhance convergence, especially in dynamic transportation environments.

These limitations do not compromise the validity of the current model but highlight opportunities for further refinement and generalization in future research.

CRediT authorship contribution statement

Nandini: Formal analysis, Funding acquisition, Writing – original draft, Data curation. **Kapil Kumar**: Visualization, Data curation, Writing – original draft, Formal analysis, Methodology. **Mukesh Kumar Sharma:** Writing – original draft, Supervision, Formal analysis, Writing – review & editing, Validation, Funding acquisition, Conceptualization, Visualization, Methodology, Data curation. **Anirudh Kumar Bhargava**: Visualization, Data curation, Funding acquisition, Project administration. **Kailash Dhanuk**: Formal analysis, Funding acquisition, Resources, Data curation. **Tarun Kumar**: Software, Conceptualization, Validation, Data curation, Formal analysis.

Author statement

- **Nandini:** Participated in the study methodology's creation and application. had a key part in the interpretation of the data and the data analysis.
- **Tarun Kumar:** Conceptualized the study, developed the theoretical framework, and led the writing of the original draft. Kumar also played a significant role in revising the manuscript critically for important intellectual content.
- **Kapil Kumar:** Contributed to the literature review and the drafting of the discussion section. Kumar also provided substantial contributions to the conceptual framework and helped refine the research questions and hypotheses based on the literature review.
- **Kailash Dhanuk:** contributed to the methodology, including design and analysis. Dhanuk was also involved in interpreting the results, as well as assisting in the writing and editing of the manuscript.
- **Anirudh Kumar Bhargava:** Contributed to the literature review and the drafting of the discussion section. Kumar also provided substantial contributions to the software analysis.
- **M.K. Sharma:** Oversaw the general plan and direction of the research. Conceptualization, Original Draft, Result Analysis, Methodology, ReSupervised the research's logistical and administrative components and helped create and edit the manuscript.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

To obtain more information about the data used in this study, please feel free to reach out to the authors. We are dedicated to offering further explanations and details to enhance the comprehension and utilization of our research outcomes.

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